The Bauschinger effect in cyclically deformed niobium single crystals

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The Bauschinger effect has been studied in niobium single crystals cyclically deformed to saturation at small strain amplitudes and at different temperatures. The parameters used to characterize the Bauschinger effect have been measured from the saturation hysteresis loops. The fatigued work-hardened crystals have been modelled as a composite consisting of a hard deformable phase (the regions of high dislocation density) embedded in a soft matrix (the regions of low dislocation density). The values of the mean internal stress in the matrix predicted by this simple composite model are in good agreement with those measured experimentally, using a modification of the method originally proposed by Cottrell to separate the lattice friction and internal stress components in reverse deformation.

1. Introduction

The Bauschinger effect is revealed by the fact that, after some deformation in a forward direction, the reverse flow stress is generally lower than the forward flow stress (permanent softening), and the stress–strain curve in the reverse direction shows a gradual yielding (transient softening). The most relevant studies of the Bauschinger effect have been devoted to investigating the permanent softening in two-phase materials consisting of a plastic matrix and a dilute dispersion of non-deformable hard inclusions [1, 2]. In this case, the permanent softening is due to the long-range internal stresses developed because of the compatibility requirement between the deformation of the matrix and the inclusions. By contrast, the transient softening has a more general nature since it has been associated with the statistics of the dislocation movement during reverse flow, that is, a dislocation moving forward sees a harder array of obstacles than it sees immediately after its motion is reversed.

There have been few studies of the Bauschinger effect in single crystals containing only one phase, presumably because in these materials the Bauschinger effect is less significant than in the above-mentioned two-phase materials. In addition, it seems that there is not a generally accepted explanation for the Bauschinger effect in one-phase materials, in contrast with the large amount of theoretical and experimental work carried out in dispersion-hardened alloys.

In the present work the Bauschinger effect has been studied in niobium single crystals cyclically deformed to saturation at small strain amplitudes. The analysis of the results has been carried out following mainly the ideas of Pedersen *et al.* [3], developed to explain the Bauschinger effect in pure copper. These authors extended the theory for two-phase materials to the study of Stage II of the flow stress of pure copper in unidirectional deformation by using a simple composite model, in which the hard phase is deformable and it consists of regions of high dislocation density embedded in a soft matrix of low dislocation density.

2. Experimental procedure

The niobium single crystals were grown by the electron-beam zone-melting technique. The total interstitial content of the crystals was about 50 p.p.m. by mass. They had a crystallographic orientation [$\bar{1}$ 4 7] which is near the centre of the unit stereographic triangle. Details of specimen preparation can be found elsewhere [4]. The ratio of the cylindrical gauge length to the specimen diameter was 3, in order to avoid buckling in compression.

The specimens were deformed cyclically in tension–compression at constant total strain amplitude until saturation was reached, that is, until the shape of the hysteresis loop remained practically unchanged with further cyclic deformation. The tests were carried out at four different temperatures between 250 and 400 K, and at a constant total strain rate of $6.0 \times 10^{-4}\,\text{sec}^{-1}$. The shear plastic strain amplitudes at saturation ranged between 2.0×10^{-3} and 10^{-2}.

The parameters used to characterize the Bauschinger effect were measured once saturation had been reached. Fig. 1 shows a typical hysteresis loop at saturation. Next to the loop, the stress–strain curves after reversal have been plotted to illustrate more clearly the Bauschinger effect. The difference in stress between the forward tensile curve and the reverse compressive curve measured at the same cumulative strain is the permanent softening in compression, $\Delta\tau_c$, while the difference in stress between the now forward compressive curve and the reverse tensile curve is the permanent softening in tension, $\Delta\tau_t$. The reason for measuring these two parameters instead of only one,

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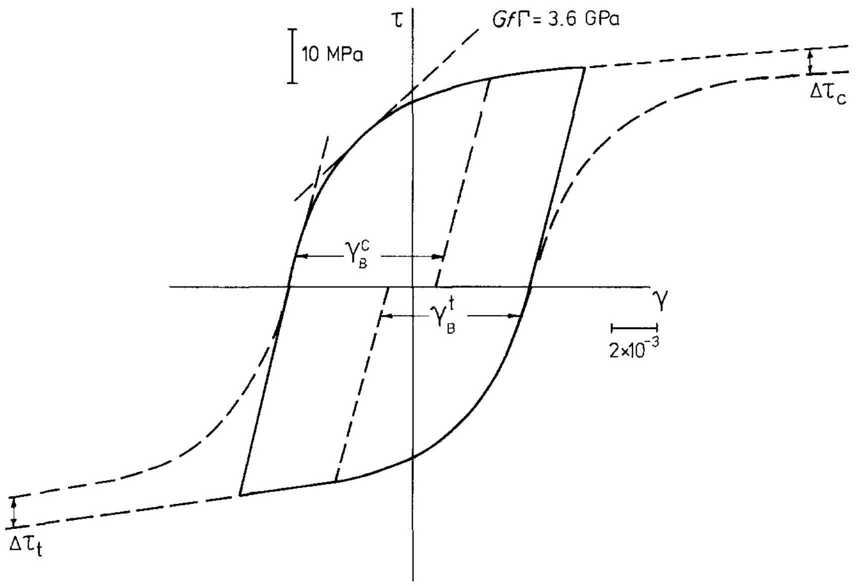

Figure 1 Typical shape of the hysteresis loop at saturation at room temperature, showing the parameters measured.

as is usually done, is because the hysteresis loop is slightly asymmetrical; the peak stress in compression is higher than in tension. This asymmetry is typical of bcc metals and is related to the asymmetry of the lattice friction stress of the screw dislocations in a bcc lattice. It changes in magnitude and sign when the crystallographic orientation of the specimen varies from [0 0 1] to [0 1 1] through the unit stereographic triangle [4, 5]. The influence of this asymmetry on the permanent softening was eliminated by taking as the permanent softening, $\Delta\tau$, the average value of $\Delta\tau_c$ and $\Delta\tau_t$.

The gradual yielding of the reverse curve has been characterized by measuring the Bauschinger strains $\gamma_B^{(c)}$ and $\gamma_B^{(t)}$ defined as the minimum reverse plastic shear strains in compression and tension, respectively, which are needed in order to reach the same slope of the hysteresis loop in the reverse direction, that is, the same plastic strain rate (see Fig. 1). If the hysteresis loops were completely symmetrical these two parameters would have the same value. However, $\gamma_B^{(c)}$ and $\gamma_B^{(t)}$ are slightly different because of a very small asymmetry in the shape of the tensile and compressive parts of the hysteresis loops, so that the average γ_B, of $\gamma_B^{(c)}$ and $\gamma_B^{(t)}$, has been taken as the Bauschinger strain.

Throughout this paper, the symbols τ and γ refer to the shear stress and plastic shear strain, respectively, acting on the plane $(1\bar{1}0)$ which has been found to be the slip plane in tension [4]. The slip plane in compression was slightly different, but the Schmid factors for both slip planes had practically the same value.

3. Results

The hardening which takes place during the cyclic deformation is illustrated in Fig. 2 for specimens deformed at room temperature at three different total strain amplitudes. The hardening of the specimen deformed at the smallest strain amplitude is negligible: this amplitude is inside the plateau region of the cyclic stress–strain curve of bcc crystals which is characterized by the absence of hardening [4–7]. The other two strain amplitudes correspond to Region 4 of the cyclic stress–strain curve where appreciable

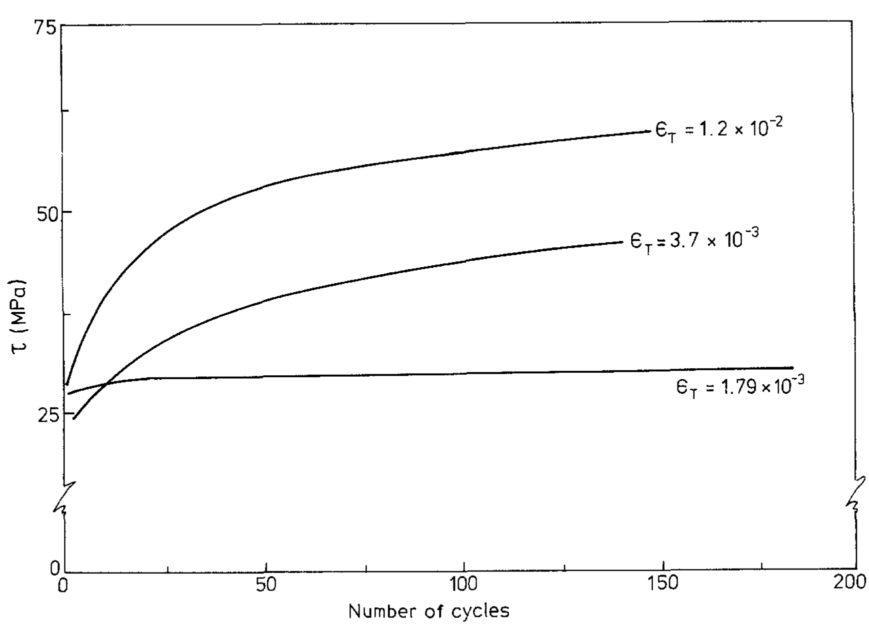

Figure 2 Cyclic hardening curves at three different strain amplitudes.

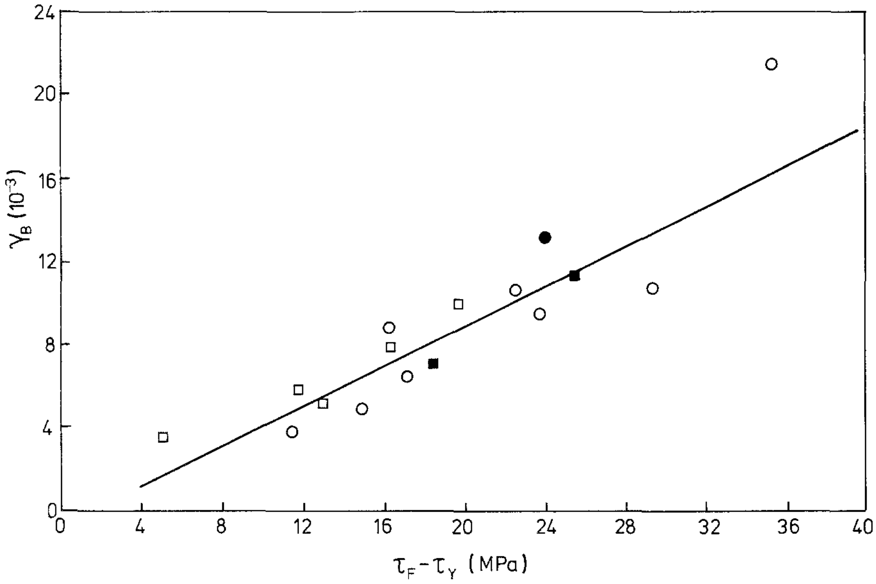

Figure 3 Variation of the Bauschinger strain with the amount of cyclic hardening for specimens deformed at different strain amplitudes and temperatures. T = (■) 250 K, (○) 295 K, (□) 354 K, (●) 400 K.

hardening occurs [4]. Both the Bauschinger strain and the permanent softening increased with the amount of hardening, $\tau_F - \tau_Y$, where τ_F is the average shear peak stress and τ_Y is the shear stress corresponding to the lower yield point in tension. A plot of γ_B against $\tau_F - \tau_Y$ at saturation is shown in Fig. 3. It can be seen that a linear relationship between γ_B and $\tau_F - \tau_Y$ represents well the experimental results.

The permanent softening was measured only after having deformed the specimens to saturation so that some work-hardening had occurred, because during the first cycles the slope of the hysteresis loops (at the points of strain reversal) was zero and, therefore, there was no permanent softening. The same happened throughout all the complete fatigue life for specimens deformed at amplitudes corresponding to the plateau region of the cyclic stress–strain curve.

One parameter commonly used to characterize the magnitude of the Bauschinger effect is the Bauschinger effect parameter (BEP), defined by

$$\mathrm{BEP} \equiv \frac{\Delta\tau}{\tau_F - \tau_Y} \tag{1}$$

The values obtained for the BEP have been plotted in Fig. 4 as a function of $\tau_F - \tau_Y$ at saturation. It can be noticed that the BEP is nearly constant and independent of temperature in the interval of strain amplitudes and temperatures studied.

4. Discussion

The dislocation substructure developed during the cyclic deformation of metals consists frequently of an inhomogeneous distribution where there are alternating regions of high and low dislocation density, respectively. The regions of high local dislocation density will be referred to as the "lumps" and they are characterized by a flow stress τ_2, which following Pedersen *et al.* [3] can be taken as the superposition of the matrix friction stress, τ_Y, and the usual forest stress, $\phi G b \varrho^{1/2}$, that is,

$$\tau_2 = \tau_Y + \phi G b \varrho^{1/2} \tag{2}$$

where ϱ is the local dislocation density in the lumps, ϕ is a constant of about 1/3, G is the shear modulus and b is the magnitude of the Burgers vector. The term τ_Y accounts not only for the lattice friction stress but also for any contribution from solid-solution hardening.

From electron microscopy observations in bcc single crystals deformed at saturation at plastic shear strain amplitudes of about 6×10^{-3}, the local dislocation density in the lumps has been estimated to be in the range between 10^{10} and $10^{11}\,\mathrm{cm}^{-2}$ in niobium [7], molybdenum [7–9] and α–Fe single crystals [9]. Using Equation 2 with $\varrho \simeq 4 \times 10^{10}\,\mathrm{cm}^{-2}$, $G = 39.6$ GPa, $b = 2.86 \times 10^{-10}$ m and taking τ_Y to be equal to the lower yield point of the virgin crystals ($\tau_Y = 25$ MPa), one finally finds $\tau_2 \simeq 100$ MPa.

The regions of low dislocation density, which from now on will be referred to as the "matrix", will be characterized by a flow stress τ_1 which is assumed to be the result of the superposition of the matrix friction stress, τ_Y, and a contribution from hardening of the matrix due to the build-up of forest dislocations

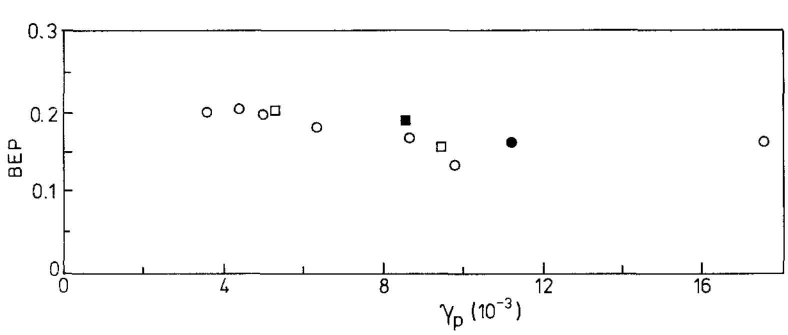

Figure 4 Variation of the Bauschinger effect parameter with the shear plastic strain amplitude at saturation. T = (■) 250 K, (○) 295 K, (□) 354 K, (●) 400 K.

during cyclic deformation, τ_f, that is

$$\tau_1 = \tau_Y + \tau_f \quad (3)$$

In the saturation stage, the fatigue-work-hardened crystals will be modelled as a composite consisting of a volume fraction f of hard but deformable lumps with a flow stress equal to τ_2 and a volume fraction $(1 - f)$ of soft matrix with a flow stress equal to τ_1. If we assume that both phases deform in parallel, then the flow stress of the lumps will be reached when $\gamma = (\tau_2/G) \simeq 2.5 \times 10^{-3}$. Therefore, for plastic shear strain amplitudes in the region of the cyclic stress–strain curve (that is, with $\gamma \geqslant 3.0 \times 10^{-3}$ for niobium at room temperature [4]) one can assume that the lumps deform plastically.

An important consequence of the composite model is the development of long-range internal stresses in the crystals in order to maintain the compatibility between the deformation of the hard and soft phases. These long-range internal stresses aid the applied stress in the lumps and oppose it in the matrix. The mean internal stress in the matrix, $\langle\tau\rangle_M$ and in the lumps, $\langle\tau\rangle_L$, are related by the condition that the mean internal stress in the composite must vanish, that is

$$(1 - f)\langle\tau\rangle_M + f\langle\tau\rangle_L = 0 \quad (4)$$

It can be shown [3] that $(\tau)_M$ can be written as

$$\langle\tau\rangle_M = fG\Gamma\gamma^T \quad (5)$$

where Γ is the Eshelby accommodation factor which varies between 0 and 1 depending on the shape of the lumps and γ^T is the effective transformation strain given by

$$\gamma^T = \gamma_2 - \gamma_1 \quad (6)$$

and it measures the incompatibility in simple plastic shear between the strain in the lumps (γ_2) and the strain in the matrix (γ_1). If it is assumed that there is no appreciable strain-hardening in the lumps then $\gamma^T = 2.5 \times 10^{-3}$. The volume fraction, f, of lumps measured at a saturation strain $\gamma_p = 6 \times 10^{-3}$ is about 0.15 [7, 8] and if $\Gamma = 0.6$ Equation 5 gives $\langle\tau\rangle_M = 9$ MPa. At the point of flow of the lumps, the mean stress in the lumps is given by

$$\langle\tau\rangle_L = \tau_2 - \tau_F \quad (7)$$

By using this expression in Equation 4 one obtains the alternative equation for the mean stress in the matrix:

$$\langle\tau\rangle_M = -\frac{f(\tau_2 - \tau_F)}{1 - f} \quad (8)$$

Thus, at $\gamma \simeq 6.0 \times 10^{-3}$, the saturation shear stress amplitude is about 40 MPa, and using $f \simeq 0.15$ together with $\tau_2 \simeq 100$ MPa, one obtains from Equation 8 $\langle\tau\rangle_M = 10$ MPa, in agreement with the value calculated above.

The permanent and transient softenings suggested by the composite model [3] are illustrated in Fig. 5 in the case where both phases, the matrix and the lumps, deform plastically. After the strain has been reversed at Point A, the matrix begins to deform plastically

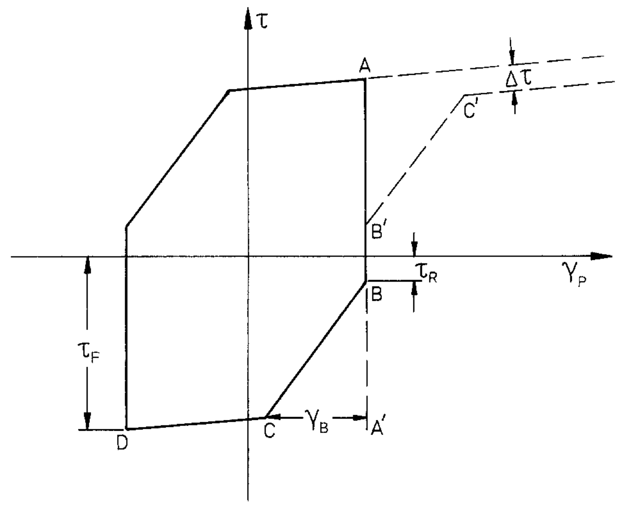

Figure 5 Shape of the hysteresis loop predicted by the composite model.

when the stress reaches Point B. In the stage BC the matrix deforms plastically while the lumps deform only elastically and the slope of the straight line is equal to $Gf\Gamma$. In the stage CD both phases deform plastically, and, in the absence of work-hardening, the line CD would be horizontal, so that the permanent softening would be zero. However, if there is some small work-hardening, then the straight line would have a slope different from zero and there would exist a small permanent softening.

With the values of f and Γ used here, the slope of the stage BC is $Gf\Gamma = 3.6$ GPa. This slope has been drawn in the hysteresis curve of Fig. 1 and it shows that the model can account fairly well for the shape of the hysteresis loop. It is not surprising that the curve is rounded in the real case, because not all the lumps have the same flow stress and some hardening of both lumps and matrix must also be allowed for.

It should be interesting to compare the value of $\langle\tau\rangle_M$ calculated from the composite model with values of the long-range back-stress obtained experimentally using the method originated by Cottrell [10] and applied more recently by Kuhlmann-Wilsdorf [11] to copper crystals. Some modifications of the method will be required here, as explained in what follows. With reference to Figs 1 and 5, the saturation shear stress in the forward direction, τ_F, can be written as

$$\tau_F = \tau_Y + \tau_f + |\langle\tau\rangle_M| \quad (9)$$

since now $\langle\tau\rangle_M$ opposes the deformation in the forward direction. Plastic deformation in the reverse direction begins at Point B (Fig. 5), when the stress reaches a value equal to τ_R which is given by

$$\tau_R = \tau'_Y + \tau_f - |\langle\tau\rangle_M| \quad (10)$$

since now the mean stress in the matrix helps the deformation in the reverse direction. Here it is assumed that τ_f is the same in the forward and reverse directions; by contrast, the lattice friction stress in the reverse direction, τ'_Y, is considered to be different from the lattice friction stress in the forward direction. The reason is that in b c c crystals the lattice friction stress is strongly dependent on the strain rate, and the plastic strain rate

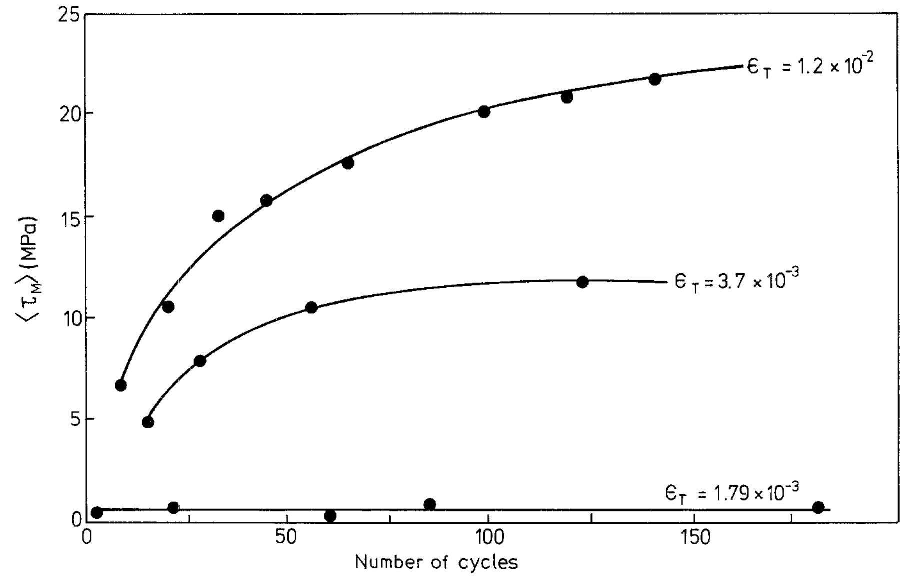

Figure 6 Variation of the mean internal stress in the matrix with the number of cycles for different strain amplitudes.

at the peak stress in the forward direction is larger than at the point where the crystal begins to yield in the reverse direction. Combination of Equations 9 and 10 provides

$$|\langle\tau\rangle_M| = \tfrac{1}{2}(\Delta\tau_F - \Delta\tau_R) \quad (11)$$

and

$$\tau_f = \tfrac{1}{2}(\Delta\tau_F + \Delta\tau_R) \quad (12)$$

where

$$\Delta\tau_F \equiv \tau_F - \tau_Y \quad (13)$$

and

$$\Delta\tau_R \equiv \tau_R - \tau'_Y \quad (14)$$

Therefore, if τ_Y and τ'_Y are taken to be equal to the values of τ_F and τ_R, respectively, of the first cycle of the virgin crystal, then by measuring τ_F and τ_R at saturation one can obtain $\Delta\tau_F$ and $\Delta\tau_R$, and finally both $\langle\tau\rangle_M$ and τ_f can be calculated from Equations 11 and 12. The build-up of $\langle\tau\rangle_M$ with the number of cycles is presented in Fig. 6 for the specimens deformed at those strain amplitudes for which the hardening curves have been shown in Fig. 2.

The back-stress at saturation for the specimen deformed at $\varepsilon_T = 3.7 \times 10^{-3}$ (giving $\gamma_p = 6 \times 10^{-3}$ at saturation) is approximately 10 MPa as calculated from the composite model.

The Bauschinger strain is related to the mean stress in the matrix by a very simple relationship; from Fig. 5 one has

$$\gamma_B \simeq \frac{\tau_F - \tau_R}{\Gamma f G} = \frac{2|\langle\tau\rangle_M|}{\Gamma f G}\left(\frac{\tau_Y - \tau'_Y}{\Gamma f G}\right) \quad (15)$$

If the term $\tau_Y - \tau'_Y$ is neglected compared with $2\langle\tau\rangle_M$, then

$$\gamma_B \simeq \frac{2|\langle\tau\rangle_M|}{\Gamma f G} \quad (16)$$

and if Equation 5 is used in this last expression, $\gamma_B \simeq 2\gamma^T$. Therefore, the Bauschinger strain is approximately equal to twice the transformation strain. The variation of γ_B with $|\langle\tau\rangle_M|$ has been plotted in Fig. 7 where a straight line represents satisfactorily the dependence of γ_B on $|\langle\tau\rangle_M|$. The slope of this straight line is $2/\Gamma f G = 0.84 \times 10^{-3}\,\mathrm{MPa}^{-1}$ and it can be seen that by using $G = 39.6$ GPa there is good quantitative agreement if $\Gamma \simeq 0.6$ and $f \simeq 0.12$, consistent with the values used before. From simple geometry and from Fig. 5 it follows that

$$\Delta\tau \simeq 2\theta\gamma_P \quad (17)$$

where θ is the hardening rate of the hysteresis loop in the stage CD where both phases deform plastically. Now the Bauschinger effect parameter, (Equation 1) can be written as

$$\mathrm{BEP} \simeq \frac{2\theta\gamma_P}{\tau_F - \tau_Y} \quad (18)$$

Furthermore, within the strain amplitude interval used in the experiments (inside the linear Region 4 of the cyclic stress–strain curve [4]), $\tau_F - \tau_Y$ is proportional to γ_P and therefore BEP $\simeq$ constant, in agreement with the plot of Fig. 4. The experimental

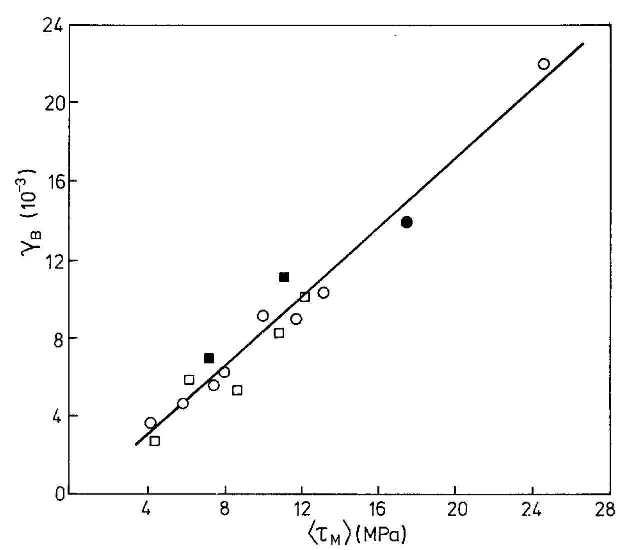

Figure 7 Plot of the Bauschinger strain against the mean internal stress for specimens deformed at different strain amplitudes and temperatures. T = (■) 250 K, (○) 295 K, (□) 354 K, (●) 400 K.

results of Fig. 3, where γ_B is roughly proportional to $\tau_F - \tau_Y$, are justified by the fact that at high strain amplitudes γ_P is nearly equal to γ_B.

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